Cognitive Science 32 (2008) 1285–1303

ISSN: 0364-0213 print / 1551-6709 online
DOI: 10.1080/03640210802477534

Measuring Model Flexibility With Parameter Space Partitioning: An Introduction and Application Example

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Received 7 August 2007; received in revised form 15 July 2008; accepted 14 September 2008

Abstract

A primary criterion on which models of cognition are evaluated is their ability to fit empirical data. To understand the reason why a model yields a good or poor fit, it is necessary to determine the data-fitting potential (i.e., flexibility) of the model. In the first part of this article, methods for comparing models and studying their flexibility are reviewed, with a focus on parameter space partitioning (PSP), a general-purpose method for analyzing and comparing all classes of cognitive models. PSP is then demonstrated in the second part of the article in which two connectionist models of speech perception (TRACE and ARTphone) are compared to learn how design differences affect model flexibility.

Keywords: Parameter space partitioning; Model comparison; Connectionist models of speech perception

1. Introduction to model comparison and evaluation

A goal of modeling in cognitive science is to infer the properties of cognition from the regularities present in experimental data. Computational models are a means by which we evaluate the accuracy of these inferences because they specify the mathematical form of the regularity. For example, we might entertain the hypothesis that memory decays according to a power function ($y = ax^{-b}$) or an exponential function ($y = a \exp(-bx)$). The purpose of such comparisons is to identify the most accurate model.

Selection of the best model is an inductive inference problem in which we generalize from specific instances (i.e., data) to all instances (i.e., model) largely by evaluating how well models can fit empirical data. This problem is ill-posed, however, because information in finite data samples (e.g., one or two experiments) is rarely sufficient to identify uniquely the

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single, true function (model) that generated the data. Random noise inherent in behavioral data further obscures the inference process. Given these obstacles, an achievable goal of modeling is to select the model, among a set of candidate models, that best approximates the cognitive process in some defined sense (Myung, 2001).

A key concept that is at the core of model selection is model flexibility, or equivalently, model complexity. It refers to the ability of a model to fit a diverse range of data patterns. A model with many parameters is more flexible than a model with few parameters. Another (often neglected) dimension of model flexibility is the model's functional form; that is, the way the model's parameters are combined to yield the model equation (Myung & Pitt, 1997). For example, the power and exponential models of retention described earlier have the same number of parameters (2), but they differ in functional form.

Model flexibility is a double-edged sword. We want a model to be flexible enough to capture the underlying cognitive process, which can be complicated; but, at the same time, the model must not be too flexible. The reason for this is that by virtue of its flexibility alone, a highly flexible model can fit a data set better than a less flexible model, even if the simpler model generated the data (Myung & Pitt, 1997; Pitt, Myung, & Zhang, 2002). The more flexible model does so by fitting idiosyncratic, random noise in the data. An implication of this situation is that choosing among a set of models based solely on a goodness-of-fit criterion—such as mean squared error, percentage of variance accounted for, or maximum likelihood (Myung, 2003)—can result in selecting an overly complex model that generalizes poorly, being unable to predict with reasonable accuracy data samples collected in future replications of that experiment.

The relation among flexibility, goodness of fit, and generalizability is illustrated in the top panel of Fig. 1. Note that goodness of fit can always be improved by increasing model flexibility (Point B vs. Point A), but at some point it will be at the cost of generalizability. It is at this point (C) along the model flexibility axis that overfitting begins to take its toll. These tradeoffs are made more tangible in the break-out boxes below the graph, which depict increasingly more complex models (lines) fitting the same data set (circles). Of the three models shown, the middle one generalizes best; it captures the trend in the data, which the one on the left fails to do, without fitting noise, which the one on the right does perfectly by passing through every data point. In short, models should not be evaluated on their goodness of fit but on their generalizability—a criterion that is considered the gold standard in statistical model selection.

Various measures of generalizability have been proposed in statistics and computer science. They include the Akaike Information Criterion (AIC; Akaike, 1973), the Bayesian Information Criterion (Schwarz, 1978), the Bayes factor (Kass & Raftery, 1995), and minimum description length (MDL: Grünwald, 2007; Grünwald, Myung, & Pitt, 2005). In each of these model selection methods, generalizability is measured by trading off goodness of fit for model flexibility. For reviews and application examples of these and other model selection methods, the reader is directed to two special issues of the *Journal of Mathematical Psychology* (Myung, Forster, & Browne, 2000; Wagenmakers & Waldorp, 2006) and an excellent survey article on the subject by Shiffrin, Lee, Kim, and Wagenmakers (this issue).

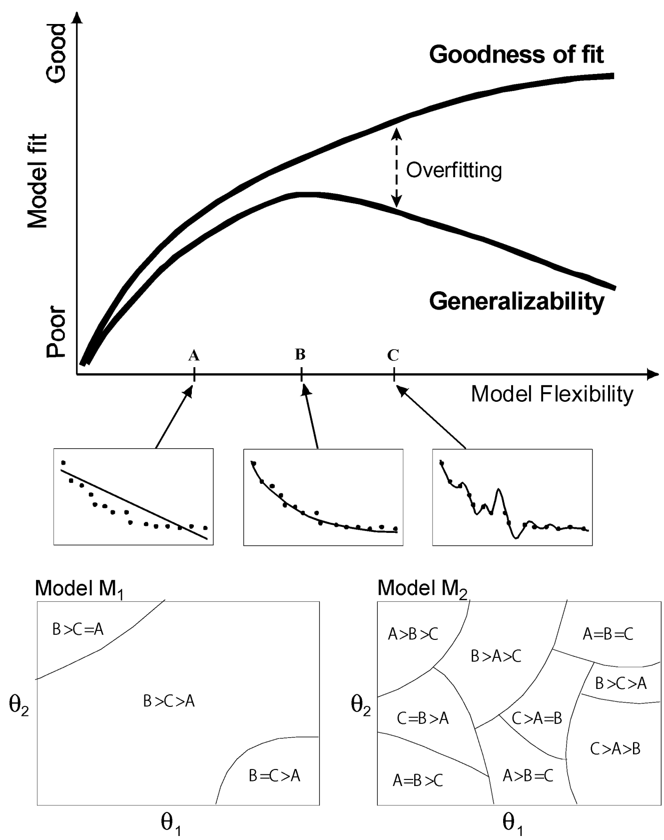

Fig. 1. Top panel: Relation among flexibility, goodness of fit, and generalizability. *Note:* The lower graphs depict the fits of three models to the same data set, increasing in flexibility from left to right (adapted from Pitt & Myung, 2002). Bottom panel: An illustration of the parameter space partitioning analysis for two hypothetical models in an experiment with three conditions: A, B, and C (adapted from Pitt, Kim, Navarro, & Myung, 2006).

2. Limitations of statistical model selection

Statistical model selection methods are designed to identify among a set of competing quantitative models the one that best approximates the underlying cognitive process, in that the model chosen accurately predicts future observations from the "true" model. As such, model selection methods are valuable tools, but their scope and usefulness are limited in two significant ways when confronted by the diverse range of models in cognitive science and the needs of cognitive scientists.

First, application of model selection methods requires that each of the models being compared be formulated as a quantitative *and* statistical model with a specified parametric family of probability distributions. By a quantitative model, we mean one with a set of free parameters that generates testable predictions at specific parameter values. Models of categorization (Nosofsky & Zaki, 2002) and retention (Rubin & Wenzel, 1996) fall into this category, but many others in cognitive science do not. Rather, they belong to a class of models that is quantitative but not statistical (i.e., no probability distributions are specified or they are intractable to derive). Examples of this class of models include connectionist models, simulation-based models such as REM (Shiffrin & Steyvers, 1997), decision field theory (Busemeyer & Townsend, 1991), and cognitive architectures such as ACTR (Anderson & Lebiere, 1998) and Clarion (Sun, 1997). For these models, statistical model selection methods are of little use.

Second, model selection methods like AIC summarize the potentially intricate relation between model and data into a single real number—a generalizability measure that is a composite score of a model's best fit to the data plus its complexity value. The first of these tells us how well the model fitted the empirical data pattern, but it is mute about how well it can fit other data patterns that might have been generated in the experiment. It may be that the model can fit equally well most data patterns or fit best data patterns that are not characteristic of human performance. Similarly, a model's complexity value is a gross measure of how many "distinguishable" data patterns a model can produce across all parameter settings (Myung, Balasubramanian, & Pitt, 2000), whereas one might be interested in only a specific range of settings.

Quite often, researchers need more than summary statistics about model performance to understand the relation between a model and data. For example, one might like to know what those additional data patterns that the model can fit actually look like, and whether they resemble human behavior. Also, does the model produce the empirical pattern over a wide range of parameter settings or over a very narrow range? Posing this question another way: How representative is the empirical pattern of the model's behavior? Questions like these probe the model–data relationship at a level of detail that is beyond the purview of statistical model selection. To answer them requires the development of a new methodology.

3. Parameter space partitioning

To fill this void, Pitt, Kim, Navarro, and Myung (2006) introduced a model analysis method dubbed parameter space partitioning (PSP). PSP finds all the data patterns that a model can produce in an experimental setting. Specifically, using an efficient search algorithm, PSP explores the entire parameter space of a model by partitioning the space into a set of disjoint regions, each of which corresponds to a different data pattern, appropriately defined by the researcher.

To illustrate how PSP works, consider an experiment in which mean response times are measured and compared across three conditions: A, B, and C. Suppose that we are interested in evaluating orderings (including equalities) among the three means such as $A > B > C$, $A > B = C$, and $B = C > A$; and further, that out of a total of 13 such qualitative data patterns, the pattern $B > C > A$ was observed in the experiment. Now consider two models, M1 and M2, each with two parameters. The bottom panel of Fig. 1 shows hypothetical PSP results for the two models. Model M1 simulates three of the 13 possible patterns. One of the three is the empirical pattern, taking up most of the parameter space, and the other two patterns are similar to the empirical pattern. The PSP analysis shows that Model M1 is not only well-constrained but also predicts the empirical pattern as its central feature. Model M2 produces 9 patterns, one of which is also the empirical pattern; but given the model's ability to simulate almost any data pattern, its account of human performance is less impressive. This impression is reinforced by the fact that the empirical pattern occupies a small region of the parameter space.

In what follows, we provide an overview of the PSP methodology, focusing on its conceptual foundations. For a thorough presentation of the technical and implementation details of the method, the reader should consult Pitt et al. (2006).

All that is required to apply PSP is that given an arbitrary choice of a model's parameter values, one must be able to derive a specific model prediction, whether it is an activation pattern, a response probability, or a reaction time. The derivation may be carried out either by means of algebraic calculations on the model equation or by directly simulating the model on computer. The first step in applying PSP is to discretize model predictions into a *finite* set of distinct data patterns. How to accomplish this, or equivalently, how to define a data pattern, is entirely up to the researcher. The solution will depend on what is reasonable given the experimental design, but can also be influenced by the choice of dependent measure and the theoretical issue being examined. Probably the most natural definition for experimentalists is one based on order-restricted constraints on model predictions (i.e., qualitatively different data patterns), as in the example in the bottom panel of Fig. 1.

Once a data pattern is defined, the next step in applying PSP is to find all of the data patterns a model can generate by searching the model's entire parameter space. For every parameter in the model, the search space increases by one dimension. Even with as few as six parameters, brute-force search methods such as grid search are too inefficient to finish in a reasonable amount of time. Pitt et al. (2006) have proposed a Markov chain Monte Carlo (MCMC) based (Gilks, Richardson, & Spiegelhalter, 1996) search algorithm that overcomes this and other limitations. The basic idea of the algorithm is that it takes random walks through the parameter space and *admits* a sequence of sampled points (i.e., parameter settings) by repeatedly applying an accept–rejection decision (e.g., do the parameter settings produce the same qualitative data pattern?) to randomly generated candidate points. The key steps of the search algorithm are as follows:

Step 0 : Initialize a set of parameter values and generate or identify a data pattern from the parameter set.

Step 1 : Sample nearby points in the parameter space according to a pre-specified MCMC sampling scheme and record the data pattern associated with each point. Some of the patterns will be the same as the present pattern, whereas others will be different. Using only the former points, estimate the shape and volume of this region.

Step 2 : Sample a nearby point just outside of the current region according to another MCMC sampling scheme.

Step 3 : Repeat Steps 1 through 2 until a proper stopping criterion is met (e.g., no new data patterns have been identified after sampling a given number of times).

The use of a MCMC sampling scheme ensures that data patterns a model can produce are discovered quickly without searching each region more than is necessary and, further, that the algorithm works efficiently even for models with many parameters. For a technical description of the algorithm and the conditions required for the algorithm to work, see Appendix A of Pitt et al. (2006). A PSP tutorial can be found at J. I. Myung's Web site.

To summarize, PSP is a model analysis tool that enables one to determine the full range of data patterns that a model can produce by varying its parameter values. As such, PSP provides a global perspective on a model's data-fitting potential, which can be obtained prior to actually conducting an experiment. Note that this property of PSP is opposite that of local analysis methods, such as goodness of fit, in which model performance is evaluated for a single set of parameter values. PSP is also versatile in that it is applicable to both statistical and non-statistical models, including connectionist models and cognitive architectures. The remainder of this article is devoted to demonstrating its application to two localist connectionist models of speech perception, to explore the consequences in performance of design differences between the models.

4. Connectionist models of speech perception

Connectionist models are strong contenders in many areas of cognitive science. In some of these fields (e.g., language, memory), multiple network architectures have been proposed to account for a set of empirical findings. For example, in the field of spoken word recognition, there are TRACE (McClelland & Elman, 1986), Merge (Norris, McQueen, & Cutler, 2000), and ARTphone (Grossberg, Boardman, & Cohen, 1997). The task of deciding among them is challenging because the behavior of even simple networks can be complex.

A primary criterion for the evaluation of connectionist models is simulation performance: Are simulation data qualitatively similar to empirical data collected in an experiment? Although simulation accuracy is a necessary requirement of any model, as discussed earlier, much more can be learned about the model's adequacy and the meaningfulness of its performance by examining the wider performance of the model in an experimental setting using PSP.

The specific question addressed in the present study was whether multilayer, hierarchical networks, like TRACE, are less flexible than those with a less-constrained architecture, like ARTphone. This possibility was prompted by empirical evidence (Vitevitch & Luce, 1999) and by a recent PSP analysis of TRACE and Merge (Pitt et al., 2006). Before discussing this work, the two models are introduced.

A diagram of ARTphone is on the left side of Fig. 2. Memory is thought to contain representations of words and smaller sub-word units, such as biphones and phonemes. Collectively, all utterances are referred to as list chunks. List chunks of the same size can inhibit each other, and larger chunks can inhibit smaller ones, which is referred to as masking. Speech is initially encoded as phonemes at the phoneme input layer, which is connected via bidirectional excitatory links to list chunks of all sizes. As speech is fed to the model, phonemes at the input layer establish a mutual excitatory feedback loop with all list chunks to which they match. This interaction is termed *resonance,* and waxes and wanes depending on the degree of match. The chunk to which the strongest resonance is established is deemed the recognized utterance.

A schematic diagram of TRACE is shown on the right side of Fig. 2. It is an interactive activation model with three layers: feature, phoneme, and word. There are excitatory

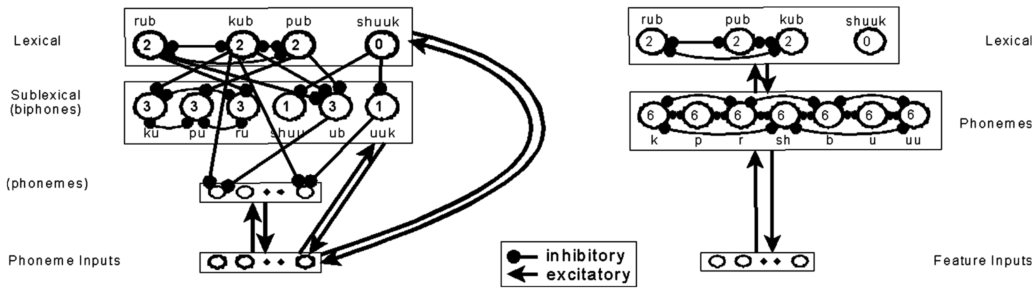

Fig. 2. Schematic illustrations of ARTphone and TRACE. *Note:* Only the most relevant links are shown. The number of these links impinging on each node is designated by the numeral inside the node.

connections between nodes in adjacent layers, and inhibitory connections between nodes within the same layer. Input is encoded in the model as features over time slices. Phonemes, and then words, are activated according to their degree of match to the input. Two salient differences between the models are the connectivity and sublexical representations. Connectivity is limited to immediately adjacent layers in TRACE, but not ARTphone, in which the phoneme input layer connects to list chunks of all sizes. Also, sublexical chunks of various sizes coexist in ARTphone, whereas there are only phonemes in TRACE.

Vitevitch and Luce (1999) were attracted to ARTphone because its connectivity seemed to provide the flexibility necessary to account for a challenging result they reported (Vitevitch & Luce, 1998). In that study, they explored the impact of phonotactic probability (frequency of phoneme co-occurrence) on word recognition. They orthogonally manipulated phonotactic probability (low vs. high) and lexical status of the utterance (word vs. nonword). Listeners had to repeat items, presented over headphones, into a microphone as quickly as possible. When the stimuli were nonwords, listeners were fastest naming the strings high in phonotactic probability. The opposite result was found with words, where listeners were fastest naming the low-probability items. The result with words was argued to be lexical in origin and due to inhibition from dense neighborhoods of words: High-probability words have many neighbors that impede recognition, whereas low-probability words have few neighbors, resulting in much less inhibition. Because nonwords do not have lexical representations, the result obtained with nonwords was thought to occur sublexically. Free from the inhibitory effects of lexical neighbors, facilitory effects of phonotactic probability could emerge, thereby causing high-probability nonwords to yield faster responses than low-probability nonwords.

Pitt, Myung, and Altieri (2007) evaluated the accuracy of this theoretical account by implementing ARTphone and performing PSP analyses across multiple testing conditions. As Vitevitch and Luce (1999) predicted, neighborhood density was the dominant force in producing the lexical effect, and substantial variation in phonotactic probability was necessary to produce the sublexical effect.

Vitevitch and Luce (1999) were concerned that TRACE's connectivity was too constraining to produce the reversal in naming speed across the two probability conditions when the stimuli changed from words to nonwords. Bidirectional excitatory connections between lexical and sublexical (e.g., phoneme) layers creates a processing dependency that would seem to restrict model performance, making it especially difficult for the model to generate one data pattern lexically with words (high density words < low density words) and the opposite data pattern sublexically with nonwords (high probability nonwords > low probability nonwords).[1] Rather, connectivity between adjacent layers would seem to reinforce, and possibly amplify, at an earlier layer (phoneme) what occurs at a later layer (lexical). The reversal in naming speed requires a dissociation of processing between layers. In contrast, the greater independence of sublexical and lexical nodes in ARTphone would seem to provide the necessary flexibility. The PSP analyses of Pitt et al. (2007) not only confirmed the latter intuition but also showed ARTphone to be very flexible with particular parameter settings.

Preliminary evidence to suggest that TRACE is less flexible than models with more independent layers was found by Pitt et al. (2006), who performed PSP analyses on Merge (Norris et al., 2002) and a bare-bones version of TRACE, created by rewiring Merge. Analyses of model performance in two experimental settings showed that Merge generated more data patterns than TRACE, suggesting that the bidirectional connectivity of TRACE does in fact constrain model performance.

The purpose of the present investigation was to explore more thoroughly the relationship between model design and model flexibility. PSP analyses were performed on ARTphone and TRACE (full-scale version) simulations in the context of the Vitevitch and Luce (1998) experimental design. Comparisons across models should answer the questions of whether TRACE is indeed more constrained in its behavior, and whether this reduced flexibility prevents it from producing the reversal in naming speed, as Vitevitch and Luce (1998) wondered.

Because the reversal in naming speed is partly attributed to differences in the size of lexical neighborhoods, PSP analyses of the models were compared across lexicons of three sizes: 4, 16, and 901 words. The size and composition of the neighborhoods differs in each lexicon. A model's sensitivity and adaptability to these differences provides another means of assessing flexibility. A more constrained model should lack the ability to produce the reversal in naming speed across lexicons because it cannot adapt to the impact of the changing contents of the lexicon. In addition, with the 901-word lexicon, we can learn how the models perform with a lexicon whose size and complexity begins to approximate that of adults.

5. Simulation details

5.1. Lexicons

The three lexicons were created so that the smaller ones were embedded in the larger ones. Our starting point was the 901-word lexicon, Biglex, which was chosen because it is the largest lexicon that jTRACE (Strauss, Harris, & Magnuson, 2007), a user-friendly version of TRACE, can use. From analyses of its neighborhood and biphone characteristics, we selected one word from a dense neighborhood (*rub*). We replaced another word in the lexicon with the string *shuuk* (rhymes with *shoot*) to create a word from a sparse neighborhood, and one that overlaps minimally with the *rub* neighborhood. The 4-word lexicon was created by combining *rub* and *shuuk* with two neighbors of *rub* (*pub, kub*). Twelve additional neighbors of *rub* were added to create the 16-word lexicon. The 16-word lexicon was created to exaggerate the difference in neighborhood size between *rub* and *shuuk* found in the 4-word lexicon.

Implementations of ARTphone and TRACE are described in detail for the 4-word lexicon. With the larger lexicons, the only difference in implementation is the number of words.

5.2. ARTphone

The version of ARTphone is similar to that described in Pitt et al. (2007), which is an expanded version of the model described in Grossberg et al. (1997). Like in Fig. 2, there are four word nodes, which are interconnected with inhibitory links to represent two levels of lexical density: low (zero neighbors; the utterance *shuuk*) and high (two neighbors; the words *rub, pub, kub*). At the sublexical layer, there are six biphones that make up the words. The total number of inhibitory and masking links impinging on each node is indicated by the numeral in each node. Chunks of the same size inhibit each other if they overlap. For words, overlap was defined in terms of biphones. For biphones, it was defined in terms of phonemes. Words also mask their corresponding biphones.

Variation in phonotactic probability across stimuli is assumed to be critical for generating the reversal in naming speed. Phonotactic probabilities were encoded in the bottom-up activation functions for the biphones and words. The functions were multiplied by an amount that corresponds to the list chunk's probability in a corpus of English. For biphones, the Phonotactic Probability Calculator was used (Vitevitch & Luce, 2004). For words, phonotactic probabilities were computed by averaging the two biphone probabilities that make up a word (Vitevitch & Luce, 1999). These values were rescaled (increasing or decreasing their range of variation) using a logarithmic function when integrated into the model so that differences in phonotactic probability influenced model performance, but did not cause erratic behavior. To perform this rescaling, two additional parameters were introduced into the model: one to rescale the probabilities for biphones (a) and another to rescale the probabilities for words (b; see Pitt et al., 2007 for additional details). Optimal values of these parameters were identified by searching the parameter space of the model. This was done separately for each lexicon.[2]

There were two word inputs to the model, *rub* and *shuuk*. The three phonemes of each word were fed to the network one at a time, each over three time units. Coarticulation was simulated by overlapping the last time unit of one phoneme with the first time unit of the next. The resonance established by these inputs at the lexical level was used as a measure of word processing, and the resonance established by the second biphone of each word (*ub* and *uuk*) was used as a measure of sublexical processing. This simulation differs from that of Pitt et al. (2007), who used as stimuli two nonwords in addition to two words. The current design was adapted from Vitevitch (2003), which is an improvement over Vitevitch and Luce (1998) in that it has the attractive feature of showing that words alone can generate the reversal; the same word can have one effect lexically and an opposite effect sublexically.

As noted earlier, PSP analyses involve repeating model simulations across the ranges of a model's parameters and then studying the data patterns that were produced. Which parameters should be varied? For ARTphone and TRACE, we varied those parameters that affected interactivity between layers because these are likely to be the most central to producing the reversal in naming speed. For ARTphone, three parameters were varied—inhibition, masking, and kappa, which is the excitation parameter responsible for resonance. For inhibition, the parameter range was 0 to.15. For masking it was 0 to.3, and for kappa it was 0 to 8.

Table 1
The nine possible data patterns in the 2 × 2 experiment of Vitevitch and Luce (1998)

	Sublexical Probability Relation		
Word Density Relation	High > Low	High = Low	High < Low
High < Low	3 Empirical	2	1
High = Low	6	5	4
High > Low	9	8	7

PSP requires that model performance be defined quantitatively. Peak resonance served as the measure of strength of evidence in favor of a biphone or word. This measure correlates negatively with reaction time, the measure of human performance in the Vitevitch and Luce (1998) experiment, where faster naming (smaller values) is indicative of more efficient processing. Therefore, to translate the empirical predictions into resonances, they must be reversed, with faster naming corresponding to greater resonance and slower naming to weaker resonance.

In the 2 × 2 experiment of Vitevitch and Luce (1998), there are nine possible data patterns if one includes ties. These are shown in Table 1, with the sublexical outcomes defined across columns and the lexical outcomes over rows. The empirical pattern is number 3. Simulation data patterns were categorized as one of these nine by comparing peak resonances of *rub* with *shuuk* and *ub* with *uuk*. To qualify as one of the patterns, all resonances had to exceed a minimum value of .2 for words and.1 for biphones. Differences less than .02 were classified as equal. Patterns that failed to meet these criteria were considered invalid and together are designated as Pattern 10. ARTphone and the PSP analyses were implemented in Matlab.

5.3. TRACE

The simulations and PSP analyses had to be modified from those performed on ARTphone to accommodate differences between the models. As shown in Fig. 2, biphones do not exist in TRACE. The only intermediate representation between the feature layer and the lexicon is the phoneme. Because of this, only the final phoneme (*b* or *k*) was used to measure sublexical activation. In addition, the lack of biphones made it impossible to encode biphone probabilities in TRACE. As a result, the two extra parameters, *a* and *b*, that were used to encode phonotactic probabilities in ARTphone were not added to TRACE. The inability to encode phonotactic probabilities makes the PSP analyses particularly interesting because they provide the opportunity to learn whether the model can compensate in some way and thereby produce the empirical pattern (3).

Network dynamics in TRACE are different from ARTphone, and this necessitated using a different quantitative definition of the Vitevitch and Luce (1998) data pattern. The effects of lexical neighborhood and other variables do not always reveal themselves as differences in peak excitation. The reason for this is that long after speech input has been fed to TRACE, activation continues to circulate through the model, causing excitation to asymptote with little, if any, subsequent decay. Word and phoneme nodes can eventually reach high and comparable levels of activation when they fully match speech input, often making this measure insensitive to neighborhood and other differences.

A more appropriate measure was needed. Pilot simulations showed that lexical influences are most salient during the rise of the node activation functions, in particular from Cycles 18 through 48. Word and final phoneme activation are underway by Cycle 18, and after Cycle 48 the activation functions assymptote. Differences between activation functions during this 31-cycle window were therefore used to define the data patterns.

The criteria used to define a data pattern as one of the nine possible were similar to that of ARTphone. The activation of *rub* was compared with *shuuk,* and *b* was compared with *k*. Word and phoneme activations had to exceed .15 from their resting levels to be counted as a valid pattern. Functions whose values differed by less than .02 were considered equal for that cycle. During the 31-cycle window, the data pattern was defined as the one that dominated across the majority of these cycles. A pattern was considered invalid (10) when a function failed to exceed threshold and when oscillating functions were generated. The original version of TRACE, written in C, was called from the PSP algorithm, implemented in Matlab.

The PSP algorithm was used to map the parameter space of TRACE. As with ARTphone, parameters were varied that were considered central to affecting the lexical and sublexical processing necessary to produce the Vitevitch and Luce (1998) data pattern: Phoneme-Phoneme inhibition, Word-Word inhibition, and Word-Phoneme excitation. The three parameters were varied across their ranges (0–1).

6. Results and discussion

The results of the PSP analyses are organized in Fig. 3 as a function of lexicon size. The bars in each graph represent the entire volume of the model's parameter space, with each slice containing the proportion of the volume occupied by the designated data pattern. Volume estimates are averages over 30 PSP runs. The patterns are stacked in ascending order. Refer to Table 1 for definitions of each pattern. Slices labeled 10 represent patterns that failed to meet the criteria to be considered a valid data pattern, usually because resonance or activation was too weak to reach threshold.

Comparison of the models when the lexicon had 4 words confirms the suspicions of Vitevitch and Luce (1999): ARTphone is more flexible than TRACE. ARTphone generated four of the nine data patterns, whereas TRACE generated only two. It is interesting to note that TRACE's two patterns are a subset of those produced by ARTphone, suggesting that TRACE's behavior is nested within ARTphone's when the lexicon is very small.

Of the patterns that ARTphone produced, the empirical pattern is dominant, occupying just over half of the parameter space (.56). A graph of ARTphone producing the reversal in naming speed is shown in the upper left portion of Fig. 4. That ARTphone can generate this data pattern over such a large range of the parameter space demonstrates its suitability in accounting for the finding. Pitt et al. (2007) obtained comparable results, although in that study ARTphone produced more patterns (8 of the 9) and the volume of Pattern 3 was smaller (.15). The difference between studies is due to the fact that three parameters were varied in

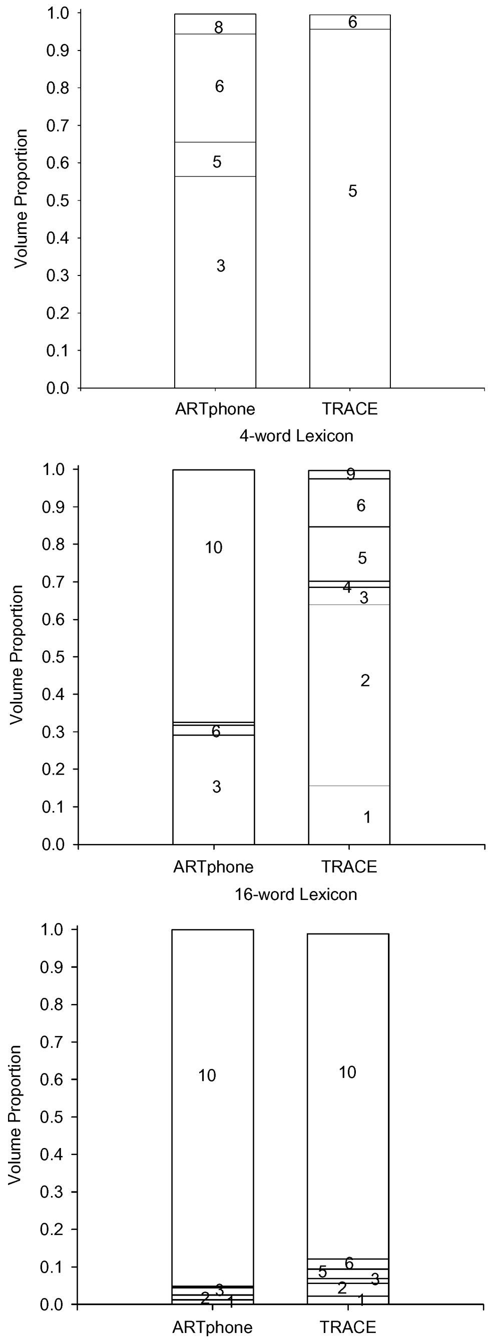

Fig. 3. The proportion of parameter space in ARTphone and TRACE occupied by each of the data patterns in the Vitevitch and Luce (1998) design.

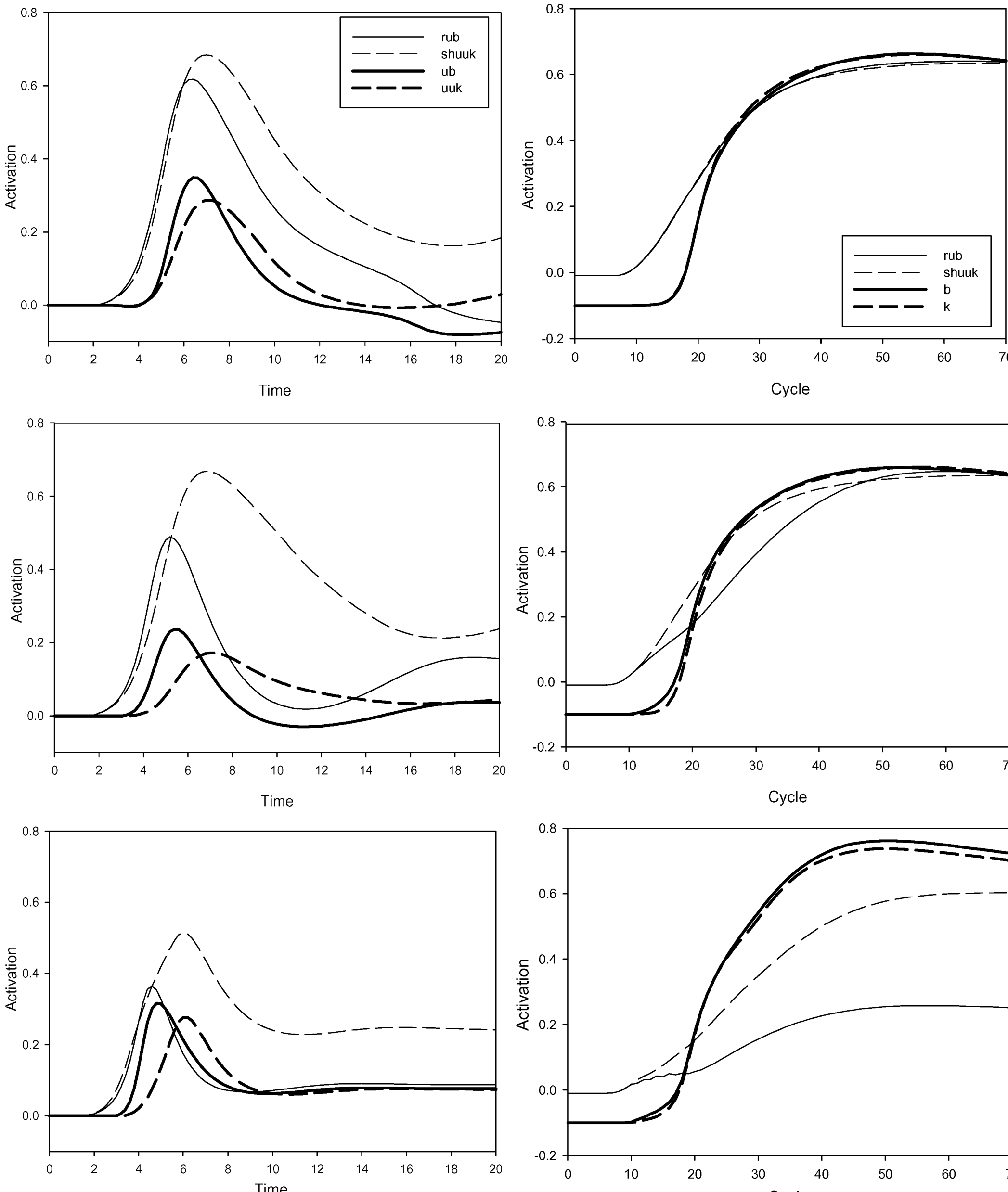

Fig. 4. ARTphone (left side) and TRACE (right side) simulations. *Note:* Output using the 4-word lexicon is in the top graph, with the 16-word and Biglex lexicons in the lower graphs. For ARTphone, the simulation is of the empirical pattern across all three lexicons. For TRACE, the simulations are of model performance with its default parameter values.

the PSP analysis of the current study instead of two, which can change the volume of a region, and the parameter settings for *a* and *b* were different.

TRACE, in contrast, could not produce Pattern 3. Instead, its volume is dominated by Pattern 5, occupying .96 of the parameter space. Pattern 5 is the null-effect pattern. It is obtained when the activation functions are comparable for the two words and for the two phonemes; the top right graph in Fig. 4 shows typical output for the model. Because TRACE has been evaluated historically using its default parameter settings, we used them in the three simulations graphed in Fig. 4 to maintain continuity with the literature. We also examined performance with values of these three parameters optimized to produce Pattern 3. With the 4-word lexicon, the activation functions are very similar to those in Fig. 4, although the phoneme functions are separated a bit more.

TRACE cannot produce the empirical pattern because of the small size of the lexicon. Inhibitory effects from lexical neighbors are the primary means by which differences in activation are generated and cycle through the model. A neighborhood of three words is too small to generate the inhibition necessary to produce the High < Low pattern lexically, let alone generate enough top-down excitation to influence sublexical processing to yield the High > Low pattern. Only at the most extreme values of the parameter responsible for top-down influences, WP, does the sublexical effect show signs of emerging.

ARTphone succeeded in producing the reversal in naming speed where TRACE failed because differences in phonotactic probabilities across words and biphones (parameters *a* and *b*) compensated for weak lexical inhibition. These extra degrees of freedom in ARTphone are the source of its greater flexibility. When phonotactic probabilities are equated across words and biphones, ARTphone no longer produces Pattern 3.

Model flexibility reverses when the neighborhood of *rub* is increased from 3 to 15 members. ARTphone again generates four patterns, but TRACE now produces 7. For ARTphone, the fivefold increase in neighborhood size causes so much masking (and, to a lesser extent, inhibition) at even low values of this parameter that biphone activation functions reach threshold only at their lowest values (<.01). When they do, the empirical pattern is generated over a significant region of the parameter space (.29). An example of ARTphone's performance in this region of the parameter space is shown in the middle graph on the left side of Fig. 4. Compared with the top graph, note how much weaker peak activation of all functions is except for *shuuk,* which, as in the 4-word lexicon, has no neighbors.

Whereas the larger neighborhood reduced the flexibility of ARTphone, it empowered TRACE with flexibility by causing *rub* to be inhibited far more than *shuuk*. The bidirectional excitatory connections between the lexical and phoneme layers caused these processing differences to spread not only to the phoneme layer, but also back up to the lexical layer. By independently varying the three parameters that control the flow of activation through the model, TRACE can produce almost any of the patterns in the experimental design. Although this includes Pattern 3 (.05 volume), it is not nearly as representative as many of the other patterns. That said, Pattern 3 can be approximated with the model's default parameter settings (middle right graph in Fig. 4), showing that it does not need to take advantage of this flexibility to mimic human behavior. Like with ARTphone, the High < Low pattern found lexically is more robust than the High > Low pattern found with the phonemes, which emerges only early in the evolution of activation (e.g., Cycles 18–30). With optimal parameter settings, the High < Low effect for words shrinks, and the High > Low effect for phonemes increases.

With Biglex, the PSP analyses of the two models resemble each other. Pattern 10 dominates the majority of the parameter space (>.85), and the region occupied by Pattern 3 is small and similar in size in both models (ARTphone = .02, TRACE = .01). Although TRACE generated two more patterns than ARTphone (6 vs. 4), the fact that the extra patterns occupy such a small amount of the total volume tempers the impact of this additional flexibility. These results shows that ARTphone's architecture is not more flexible than TRACE's, at least within the context of the Vitevitch and Luce (1998) data. TRACE generates just as many data patterns as ARTphone, including the empirical one.

Despite their structural and representational differences, the two models perform much more similarly when a more realistic lexicon is used. When the lexicon is large, it—more than differences in model architecture—governs model performance. Words act *en mass* to constrain model behavior significantly. With so many words in the lexicon, "valid" data patterns (i.e., 1–9) are found in both models only when parameter values are within a narrow range and together occupy less than.15 of the volume, which is far different from what was found with the smaller lexicons. In addition, the parameters for inhibition (and masking for ARTphone) must be very, very low. Otherwise, activation functions will not exceed threshold.

Other ways in which the large lexicon affected model performance are visible in the bottom two graphs of Fig. 4. For ARTphone, the activation functions become more peaked with a larger lexicon. In addition, the functions for *shuuk* and *uuk* are shifted later in time, probably another consequence of *shuuk* being in a sparse neighborhood. Composition of the lexicon also has temporal processing consequences in TRACE, although they are less obvious to the eye. The High < Low pattern for words emerges early (Cycle 11), and the functions diverge quickly. On the other hand, the sublexical High > Low pattern does not emerge permanently until Cycle 24, as though there are forces preventing the high probability phoneme, *b*, from rising faster than *k*. Even then, the functions diverge slowly from Cycle 24 onward. That the time course of these two effects is due to properties of the broader lexicon can be seen by comparing these functions with those in the graph above it, where the lexicon contains only two neighborhoods, one with 15 words and the other with 1. Activations functions for the phonemes start to diverge much earlier, at the same time as the words. They then converge at Cycle 30 and stay that way for the remaining cycles. In contrast to the High < Low pattern for words, which is temporally similar across Biglex and the 16-word lexicon, the emergence of the High > Low pattern for phonemes is more complex. Because the 16-word lexicon is contained within Biglex, the presence of other words is at least partially responsible for this change in the activation time course.

In the experimental literature, the sublexical High > Low pattern has also proven to be complex. It is frequently obtained when participants respond quickly, but when responses are slow or the pace of the experiment is slow, the opposite outcome (High < Low) or a null effect is found (Lipinski & Gupta, 2005; Vitevitch & Luce, 2005). Vitevitch and Luce (2005) raised the possibility that the High > Low data pattern is a time-dependent process, occurring only early in word processing. The present simulations illustrate one context in which this can be the case, but they suggest more strongly that a broader consideration of overlap in the lexicon could assist in understanding the sublexical effect.

7. Conclusion

PSP was used in this study to understand the consequences of design differences between two localist models of speech perception: ARTphone and TRACE. In the context of the Vitevitch and Luce (1998) experimental design, the analyses showed that the two models are comparable in flexibility when the simulations include a more realistically sized lexicon, generating a similar number of data patterns and with the empirical pattern occupying a similarly sized region in the parameter space. When small lexicons were used, design differences were evident, one of which was ARTphone's superior ability to generate Pattern 3 across lexicons—an indication of its greater flexibility. In some circumstances (16-word lexicon), it is clear that TRACE can exhibit a high degree of flexibility. The suspicions of Vitevitch and Luce (1999) were partly correct. ARTphone can be more flexible than TRACE, but TRACE itself is sufficiently flexible to generate the reversal in naming speed.

How are these two very different models able to produce the reversal? The challenge lies not so much in producing the lexical effect (High < Low) as it is in producing the sublexical effect (High > Low). The lexical pattern comes about in both models from greater lexical inhibition in dense than in sparse neighborhoods, resulting in the low-density word achieving a higher level of activation. The origin of the sublexical pattern is quite different in the two models because of how the lexicon interacts with sublexical layers. As shown in Fig. 2, in ARTphone, biphones are masked by words with which they overlap, which means that high-probability biphones will naturally receive greater masking than low-probability biphones. To offset this effect, phonotactic probability is explicitly encoded in biphone excitation levels so that high-probability biphones achieve a higher level of activation. Phonotactic probability differences, motivated by the data of Vitevitch and Luce (1998), are key ingredients to producing the effect in ARTphone.

In TRACE, top-down effects are facilitory, so dense neighborhoods of words boost the sublexical activation of phonemes in these same words, which means the phonemes of words from dense neighborhoods will achieve a higher level of activation than those from sparse neighborhoods, yielding the High > Low pattern. Phonotactic (or phoneme) probability does not need to be encoded directly in TRACE because it emerges from the combined influence of overlapping words in the lexicon. The sublexical pattern is another example of a "gang" effect in TRACE (McClelland & Elman, 1986). The same principle is present in ARTphone, but because top-down effects are inhibitory, the model's natural behavior is to produce the High < Low pattern sublexically when parameters for phonotactic probability (*a* and *b*) are not present. This connectivity difference between the models is likely to be important in distinguishing between them in future work.

The PSP analyses across lexicons make clear the central role of the lexicon in determining the behavior of the two models. The vastly different results across the three lexicons show that its composition can greatly affect model behavior. Compared to small lexicons, large ones squash model flexibility by limiting the effectiveness of parameter variation in generating sensible (i.e., valid) data patterns. In this respect, the lexicon functions almost as a meta-parameter.

Model flexibility is a complex issue that modelers must confront to understand the meaning and implications of their simulations. As mentioned in the introduction, a model must have enough flexibility to generate the empirical pattern of interest. Otherwise, the model is clearly inadequate. Because a successful model will eventually be applied to many testing situations, a high degree of flexibility may be necessary for the model to generate very different data patterns. There is a catch-22 here in that the high degree of flexibility needed for the model to generalize to new testing situations can lead to the model generating most or all data patterns that might be observed in a single experiment. The broad perspective that PSP provides about model behavior allows one to identify such situations, and thus understand what it means for a model to simulate a data pattern. In this regard, it is impressive that with the larger lexicons, TRACE did a reasonably good job of simulating the empirical pattern with its default parameter settings.

Another way to think about the problem of flexibility, and the implications it has for model evaluation, is as a mismatch between a model and the ability of an experimental design to test it or to discriminate between a set of models. PSP provides a means of assessing whether a model's flexibility mismatches the power of the experimental design. With such knowledge in hand, the modeler has some idea of the quality of the data and its potential to advance the field. PSP extracts much richer information about a model than statistical model selection methods, such as AIC and MDL. Information gleaned from applying PSP can help us deepen our understanding of how and why the model behaves the way it does, thereby improving modeling in the cognitive sciences.

Notes

1. *Word density* is used to refer to the manipulation of word probability because the two correlate highly and word density is most descriptive of the source of the High $<$ Low data pattern, neighborhood size.
2. Other analyses showed that ARTphone cannot produce the empirical pattern across the three lexicons with a and b, the parameters that encode phontactic probability, held constant. Such values can be found when only the 4-word and 16-word lexicons are considered. Because parameter space partitioning analyses with these settings are similar to those reported, we decided to use the optimal settings for these parameters for each lexicon. A similar comparison was performed on TRACE, and is described in the main text.

Acknowledgments

This work was supported by research Grant R01–MH57472 from the National Institute of Mental Health, National Institute of Health. Portions of this work were presented at the 2006 annual meeting of the Society for Mathematical Psychology.

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